Numerical Linear Algebra
CSCI 5646
Fall 2023

Instructor:
Rebecca Morrison
ECOT 820
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Class details:
Class meetings: T/Th 9:30 – 10:45, ECCR 105
Grader: Viveka Salinamakki, viveka.salinamakki@colorado.edu
Office hours:
RM: Mon 10:30 – 11:30a, ECOT 820, and by appointment

Materials:
Numerical Linear Algebra by Trefethen and Bau
Lecture notes (R. Morrison)
Canvas for announcements, class recordings, and grading
Slack for questions about the course and student-led discussions (See Canvas for link)

Note about email: Email should be used only for personal/individual matters, and even then it is better to come see me in person. Any emails about the class content, logistics, etc. will be posted on Slack and answered for everyone there.

Course description

This course will cover the fundamental ideas of numerical linear algebra, which, as Trefethen and Bau point out in their preface, is "really *applied* linear algebra." They go further: "Numerical linear algebra is really functional analysis, but with the emphasis always on practical algorithmic ideas rather than mathematical technicalities." In this class, we will develop some of the theory and get lots of practice with examples. Topics include the basics (matrix operations, norms, and the SVD), QR factorization and least squares, conditioning and stability, systems of equations, eigenvalues, and, time permitting, iterative methods.

Course objectives

In this course, students will learn how to:

- Perform common matrix operations, compute and manipulate norms, and become comfortable with complex matrices and vectors
- Compute the SVD by hand for small matrices, and implement the SVD on a computer
- Leverage the QR factorization for many further purposes
- Compute and compare conditioning and stability of various modern and historical algorithms
- Solve systems of equations

- Find eigenvalues
- Begin to analyze common iterative methods
- Appreciate numerical linear algebra for the "beautiful and fundamental" subject that it is!

Course outline, by week*

W1 (8/29, 8/31) Introductions, Syllabus, L1: Matrix-vector multiplication
W2 (9/5, 9/7) L2: Orthogonal vectors & matrices
W3 (9/12, 9/14) L3: Norms
W4 (9/19, 9/21) L4-5: SVD
W5 (9/26, 9/28) L6: Projectors; L7-8: QR & Gram-Schmidt
W6 (10/3, 10/5) L10: Householder, L11: Least squares
W7 (10/10, 10/12) L12: Condition numbers
W8 (10/17, 10/19) L13: Floating point, L14-15: Stability, (L16-19: S&C of some algorithms)
W9 (10/24, 10/26) L20: Gaussian Elimination, L21: Pivoting
W10 (10/31, 11/2) L22: Stability of GE, L23: Cholesky
W11 (11/7, 11/9) L24-25: Eigenvalue problems & algorithms, L26-27: Reductions, Rayleigh quotient
W12 (11/14, 11/16) L28-29: QR with & without shifts, L30-31: Other algorithms, SVD
W13 (11/21, 11/23) Fall break
W14 (11/28, 11/30) Flex week
W15 (12/5, 12/7) L32-34: Overview of iterative methods, Arnoldi; L36-37: Lanczos, Gauss quadrature
W16 (12/12, 12/14) L38: Conjugate gradients, L40: Preconditioning

*Note that this schedule is approximate and subject to modifications.

Course work and grading

Grades will be determined based on homeworks.

- **Homework:** Homeworks will be assigned approximately every two weeks. You will be asked to upload a pdf (of handwritten exercises, typed exercises, and/or a Jupyter notebook or other code). You may complete homework assignments in groups, but please turn in your own work, and make sure you understand each step. If you are very sure that the homework has been misgraded, you may contact Viveka directly (and cc me). Otherwise, please accept the grade and try to understand what you could do better or make clearer next time.
  - **Homework drop:** The last homework is optional and, if completed, its score will replace the lowest homework grade.
  - **Late policy:** Because of the large class size, late homework will not be accepted (in general).

- **Extra credit:** Turn in a nice, clean, edited draft of class notes, in LaTeX. This must be emailed to me by the start of the following Tuesday class period. We will have a sign-up sheet if needed (i.e., if I start receiving lots of duplicates). Every (acceptable) day of notes will add 1% to your final grade.
- **More extra credit:** If you would like your typed, LaTeX'd homework solutions to be considered for extra credit, please flag Viveka. If you get a good score and show all your work, we will take the LaTeX file and give 20 points extra credit on that assignment.

Note: You may choose whether or not to attend class, and I aim to make class worth your time. If you do attend, I expect you to arrive on time and stay until the end.